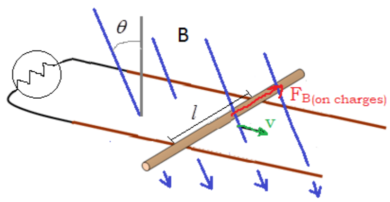
Homework 11 (Solutions): Motors/Generators

Problem 1. A 10Ω light bulb is connected to the sliding bar via conducting rails. And the ℓ = 20cm bar is immersed in a 5T magnetic field, making an angle θ = 20° with the vertical.



(a) How fast and in which direction must you move the bar if you want the current to circulate CCW with magnitude 2.5A?

Must move bar to the right, as then the electromotive (magnetic) force $\mathbf{F}_{\mathbf{B(on\ charges)}} = q\mathbf{v}\times\mathbf{B}$ will point upwards, moving the current up the wire, and therefore counterclockwise.

The effective emf generated by this force is $\xi = B\ell v\cos\phi$. So then applying Ohm's law, we demand:

$$I = \frac{\xi_{eff}}{R}$$

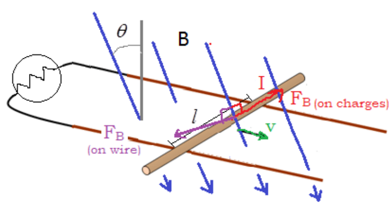
$$I = \frac{Blv\cos\theta}{R}$$

$$2.5\text{A} = \frac{(5\text{T})(0.20\text{m})v\cos 20^\circ}{10\Omega}$$

$$v = \frac{(2.5\text{A})(10\Omega)}{(5\text{T})(0.20\text{m})\cos(20^\circ)} = 26.6\,\text{m/s}$$

(b) What force **F** must you exert (this is equal/opposite to the force the magnetic field exerts on the wire, $\mathbf{F}_B$)?

The 2.5A current just induced will give rise to another force on the wire, $\mathbf{F}_{\mathbf{B(on\ wire)}} = \ell\mathbf{I}\times\mathbf{B}$. And this force is the one we must counter:



$$\begin{aligned} F_{you} &= F_{B(on\,wire)} \\ &= lIB\sin\varphi \\ &= (0.20\text{m})(2.5\text{A})(5\text{T})\sin(90^\circ) \\ &= 2.5\text{N} \end{aligned}$$

Note this force is perpendicular to B (always is), and so that gives it a 20° angle below the horizontal. Presumably, the the rails would counter the vertical component of the force (like a normal force), and so all you'd be responsible for would be the horizontal component,

$$\begin{aligned} F_{you} &= F_{B(on\,wire)}\cos\varphi \\ &= (2.5\text{N})\cos(20^\circ) \\ &= 2.35\text{N} \end{aligned}$$

(c) Let's take a look at energy conservation. The power you exert should equal the power dissipated by the lightbulb. What power (**F**·**v**) do you exert? What power (i^2R) does the lightbulb dissipate? Are these the same?
Our power is:

$$\begin{aligned} P_{you} &= \mathbf{F}_{you}\cdot\mathbf{v} \\ &= (2.35\text{N})(26.6\text{m/s}) \\ &= 62.5\,\text{W} \end{aligned}$$

And the power dissipated across the lightbulb is:

$$\begin{aligned} P_{dissipated} &= i^2R \\ &= (2.5\text{A})^2(10\Omega) \\ &= 62.5\,\text{W} \end{aligned}$$

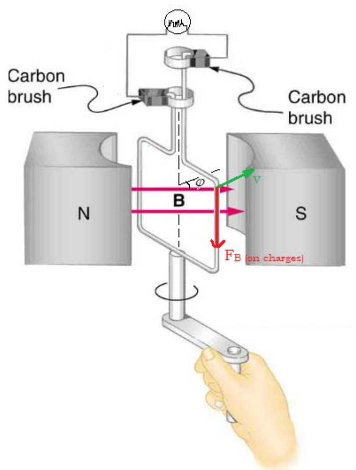
So power is conserved. Yay!

(d) And now let's take a look at the magnetic field's net power. Magnetic fields are supposed to do no net work (or power). And here the magnetic field seems to exert two different powers: one is $P_{emf} = I\xi_{effective}$ (basically IΔV from the circuits stuff we studied before) and the other is $P_{mechanical}$ = $\mathbf{F}_B\cdot\mathbf{v}$. What are these powers? And what is the net power the magnetic field exerts? Is it zero?
So we calculate:

$$
\begin{aligned}
P_B &= P_{mechanical} + P_{emf} \\
&= \mathbf{F}_{B(on\,wire)} \cdot \mathbf{v} + I\xi_{emf} \\
&= -(2.35\text{N})(26.6\text{m/s}) + (2.5\text{A})(Blv\cos\varphi) \\
&= -62.5\,\text{W} + (2.5\text{A})(5\text{T})(0.20\text{m})(26.6\text{m/s})\cos(20^\circ) \\
&= -62.5\,\text{W} + 62.5\text{W} \\
&= 0\text{W}
\end{aligned}
$$

As required. Note the negative sign on the first term is due to the fact that the magnetic force on the wire and the velocity are pointing in opposite directions (so the dot product gives – sign).

Problem 2. Now our 10Ω light bulb is connected to an N = 75 square loop (side lengths ℓ = 20cm) being rotated in a 5T magnetic field, which presently makes a 20° angle w/r to the loop.



(a) What frequency of rotation, f, and in what direction (CW, CCW) must you rotate the loop if you want the current to pass rightwards through the lightbulb with magnitude 2.5A?
Need to rotate counter-clockwise. Then we'll get the situation above, which will make the current ultimately pass rightward through the resistor. The frequency of rotation necessary is ultimately given by Ohm's law:

$$I = \frac{\xi_{eff}}{R}$$

$$2.5\text{A} = \frac{NBA\omega \sin\varphi}{R}$$

$$2.5\text{A} = \frac{(75)(5\text{T})(0.20\text{m})^2 \omega \sin 20^\circ}{10\Omega}$$

$$\omega = \frac{(2.5\text{A})(10\Omega)}{(75)(5\text{T})(0.20\text{m})^2 \sin 20^\circ} = 4.87\,\text{rad/s}$$

And this corresponds to a frequency:

$$f = \frac{\omega}{2\pi} = 0.77\text{Hz}$$

(b) What torque **τ** must you exert (you need to calculate the $\mathbf{F}_B$ force on the front wire and back wire, then the torque each of these exerts about the central axis, and then sum to get the net torque, $\boldsymbol{\tau}_B$; your torque is equal/opposite to this)?
So the magnetic field will exert a counter-torque on the wire(s). And this is the one we must counter:

$$\begin{aligned}
\tau_{you} &= \tau_{B(\text{on wire})} \\
&= \mu B \sin\varphi \\
&= (NIA)(B)\sin 20^\circ \\
&= (75)(2.5\text{A})(0.20\text{m})^2 (5\text{T})\sin 20^\circ \\
&= 12.8\,\text{Nm}
\end{aligned}$$

(c) Let's take a look at energy conservation. The power you exert should equal the power dissipated by the lightbulb. What power (**τ·ω**) do you exert? What power (i^2R) does the lightbulb dissipate? Are these the same?
Well,

$$\begin{aligned}
P_{you} &= \boldsymbol{\tau}_{you} \cdot \boldsymbol{\omega} \\
&= (12.8\text{Nm})(4.87\,\text{rad/s}) \\
&= 62.5\,\text{W}
\end{aligned}$$

And power dissipated is:

$$\begin{aligned}
P_{dissipated} &= i^2 R \\
&= (2.5\text{A})^2 (10\Omega) \\
&= 62.5\text{W}
\end{aligned}$$

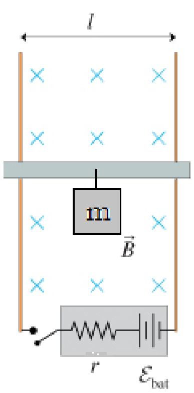
So these match ☺.

(d) And now let's take a look at the magnetic field's net power. Magnetic fields are supposed to do no net work (or power). And here the magnetic field seems to exert two different powers: one is $P_{emf} = I\xi_{effective}$ and the other is $P_{mechanical} = \boldsymbol{\tau}_B \cdot \boldsymbol{\omega}$. What are these powers? And what is the net power the magnetic field exerts? Is it zero?

These are:

$$\begin{aligned} P_B &= P_{mechanical} + P_{emf} \\ &= \boldsymbol{\tau}_{B(on\,wire)} \cdot \boldsymbol{\omega} + I\xi_{emf} \\ &= -(12.8\text{N})(4.87\text{rad/s}) + (2.5\text{A})(NBA\omega \sin\varphi) \\ &= -62.5\,\text{W} + (2.5\text{A})(75)(5\text{T})(0.20\text{m})^2(4.87\text{rad/s})\sin(20^\circ) \\ &= -62.5\,\text{W} + 62.5\text{W} \\ &= 0\text{W} \end{aligned}$$

Problem 3. A battery with internal resistance r = 25mΩ, and potential difference ξ_{bat} is connected to an ℓ = 20cm slide rail supporting a weight of m = 50kg. And this is immersed in a magnetic field B = 2T.



(a) What force must the magnetic field exert on the bar to raise it at rate 2m/s (speed makes no difference in this calculation)?

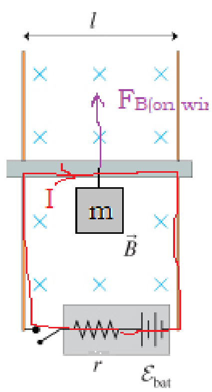
The magnetic force must counter gravity's force:

$$\begin{aligned} F_{B(on\,wire)} &= mg \\ &= (50\,\text{kg})(9.8\,\text{N/kg}) \\ &= 490\text{N} \end{aligned}$$

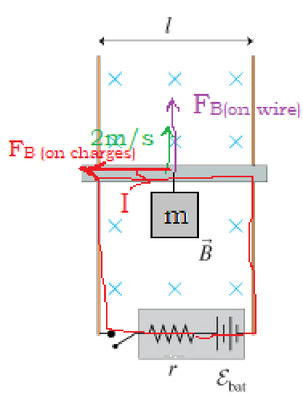
(b) What current must be flowing through the bar to engender this force? Is the battery hooked up to make the current flow in the proper direction to produce the desired upwards force?
Well,

$$F_{B(on\,wire)} = 490\text{N}$$
$$lIB\sin\varphi = 490\text{N}$$
$$(0.20\text{m})I(2\text{T})\sin 90^\circ = 490\text{N}$$
$$I = \frac{490\text{N}}{(0.20\text{m})(2T)} = 1.23\,\text{kA}$$

And battery is in proper direction, as it would produce a current going rightward across wire, which would result in upward magnetic force.



(c) What must ξ_{bat} be to give rise to this current?
Here we use KVL, not forgetting that going across the wire we will encounter the effective potential difference ($\xi_{eff} = B\ell v\cos\phi$) the battery exerts on the charges as a result of their velocity in the upward direction:



$$
\begin{aligned}
&+\xi_{batt} - ir - \xi_{eff} = 0 \\
&\xi_{batt} - (1.23\text{kA})(25\text{m}\Omega) - Blv\cos\varphi = 0 \\
&\xi_{batt} = (1.23\text{kA})(25\text{m}\Omega) + (2\text{T})(0.20\text{m})(2\text{m/s})\cos 0^\circ \\
&\qquad = 31.5\,\text{V}
\end{aligned}
$$

(d) Now let's take a look at energy (power) conservation again. What is the power supplied by the battery? What is the power absorbed by the resistor? What is the mechanical power absorbed by the weight ($P_{mechanical}$ = dPE_g/dt = $mg \cdot dh/dt$ = mgv)? Does the power supplied equal the power absorbed?

Power supplied by battery is:

$$
\begin{aligned}
P_{batt} &= i\xi_{batt} \\
&= (1230\text{A})(31.5\text{V}) \\
&= 39\text{kW}
\end{aligned}
$$

Power absorbed by resistor is:

$$
\begin{aligned}
P_{dissipated} &= i^2 r \\
&= (1230\text{A})^2(0.025\Omega) \\
&= 38\,\text{kW}
\end{aligned}
$$

And power absorbed by weight is:

$$
\begin{aligned}
P_{weight} &= mgv \\
&= (50\,\text{kg})(9.8\text{N/kg})(2\text{m/s}) \\
&= 1\text{kW}
\end{aligned}
$$

So energy is conserved.

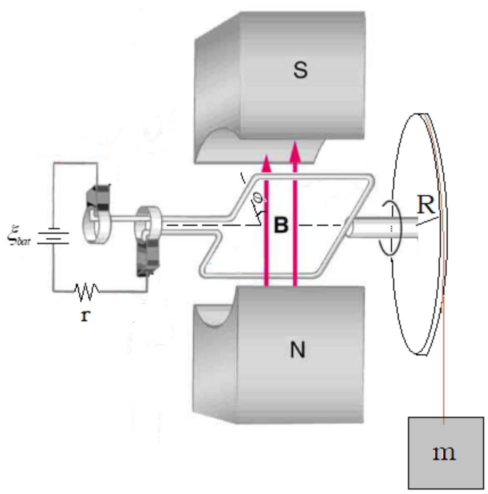
(e) And now let's check that the magnetic field does no net work (power). The magnetic field exerts two powers, again. It exerts a mechanical power $P_{mechanical}$ = $\mathbf{F}_B \cdot \mathbf{v}$, and also $P_{backemf}$ = $I \cdot \xi_{effective}$. Do these two add up to zero?

Checkin it out,

$$
\begin{aligned}
P_B &= P_{mechanical} + P_{emf} \\
&= \mathbf{F}_{B(on\,wire)} \cdot \mathbf{v} + I\xi_{emf} \\
&= -(490\text{N})(2\text{m/s}) + (1230\text{A})(Blv\cos\varphi) \\
&= -980\,\text{W} + (1230\text{A})(2\text{T})(0.20\text{m})(2\text{m/s})\cos(0^\circ) \\
&= -980\,\text{W} + 980\text{W} \\
&= 0\text{W}
\end{aligned}
$$

Yes!

Problem 4. Now consider a different, more economical motor. So say we go back to our N = 75 turns, ℓ = 20cm square loop, immersed in magnetic field B = 5T, and presently making 20° angle with the loop. Let's also take the internal resistance of our battery to be r = 25mΩ. And finally, let's say that we're trying to use our battery to generate a torque sufficient to lift this m = 50kg mass via pulley with radius R = 5cm.



(a) What torque must the magnetic field exert on the loop to raise the bar at an angular rate ω = 2rad/s (angular velocity makes no difference in this calculation)?

The torqe it must exert is just equal to the torque gravity is exerting on the mass:

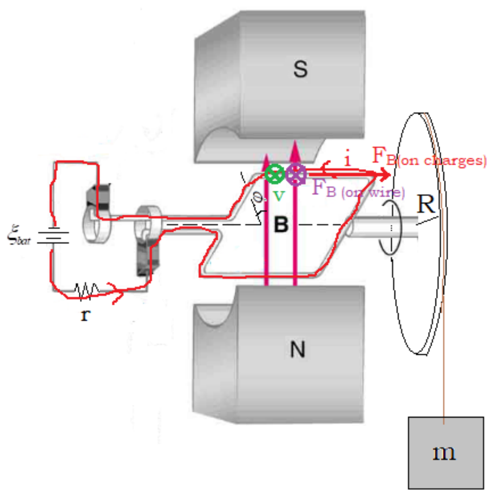
$$\begin{aligned}\tau_{B(on\,wire)} &= mgR \\ &= (50\,\text{kg})(9.8\,\text{N/kg})(0.05\text{m}) \\ &= (490\text{N})(0.05\text{m}) \\ &= 25\text{Nm}\end{aligned}$$

(b) What current must be flowing through the loop to engender this torque? Is the battery hooked up to make the current flow in the proper direction, to produce the desired CCW torque?

We need,

$$\tau_{B(on\,wire)} = 25\,\text{Nm}$$
$$\mu B \sin\varphi = 25\text{Nm}$$
$$(NIA)B\sin\varphi = 25\text{Nm}$$
$$(75)I(0.20\text{m})^2(5\text{T})\sin 20^\circ = 25\text{Nm}$$
$$I = \frac{25\text{Nm}}{(75)(0.20\text{m})^2(5\text{T})\sin 20^\circ} = 4.9\text{A}$$

And it is hooked up in the correct direction, because I was too lazy to make it wrong. And it we can see that because if the current is going to the left along the top wire, then the force on the wire $F_{B\,(wire)}$ will point into the page, which would rotate the wheel counter-clockwise, viewed from left, which would pull the weight upward.



(c) What must ξ_{bat} be to give rise to this current?
Referring to the same diagram, we'll note that there will be an emf $\xi_{eff} = NBA\omega\sin\phi$ opposing the battery, due to $F_{B\,(on\ charges)}$. Taking account of this, and applying KVL to the loop, we have:

$$+\xi_{batt} - ir - \xi_{eff} = 0$$
$$\xi_{batt} - (4.9\text{A})(0.025\Omega) - NBA\omega\sin\varphi = 0$$
$$\xi_{batt} = (4.9\text{A})(0.025\Omega) + (75)(5\text{T})(0.20\text{m})^2(2\text{rad/s})\sin 20^\circ$$
$$= 10.4\,\text{V}$$

(d) Now let's take a look at energy (power) conservation again. What is the power supplied by the battery? What is the power absorbed by the resistor? What is the mechanical power absorbed by the weight ($P_{mechanical} = dPE_g/dt = mgv = mg\omega R$)? Does the power supplied equal the power absorbed?
Power supplied by battery is:

$$\begin{aligned}P_{batt} &= i\xi_{batt}\\ &= (4.9\text{A})(10.4\text{V})\\ &= 51\text{W}\end{aligned}$$

Power absorbed by resistor is:

$$\begin{aligned}P_{dissipated} &= i^2 r\\ &= (4.9\text{A})^2(0.025\Omega)\\ &= 0.6\text{W}\end{aligned}$$

And power absorbed by weight is:

$$\begin{aligned}P_{weight} &= mg\omega R\\ &= (50\,\text{kg})(9.8\text{N/ kg})(2\text{rad/s})(0.05\text{m})\\ &= 50\text{W}\end{aligned}$$

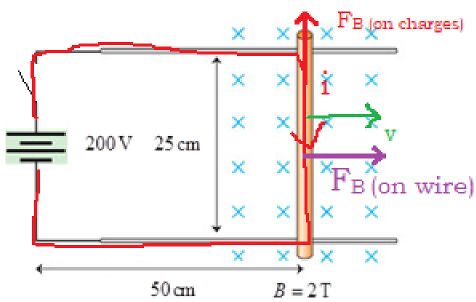
So energy is conserved, within rounding errors.

(e) And now let's check that the magnetic field does no net work (power). The magnetic field exerts two powers, again. It exerts a mechanical power $P_{mechanical} = \boldsymbol{\tau}_B \cdot \boldsymbol{\omega}$, and also $P_{backemf} = I \cdot \xi_{effective}$. Do these two add up to zero?
Checkin it out,

$$\begin{aligned}P_B &= P_{mechanical} + P_{emf}\\ &= \boldsymbol{\tau}_{B(on\,wire)} \cdot \boldsymbol{\omega} + I\xi_{emf}\\ &= -(25\text{N})(2\text{rad/s}) + (4.9\text{A})(NBA\omega \sin\varphi)\\ &= -50\,\text{W} + (4.9\text{A})(75)(5\text{T})(0.20\text{m})^2(2\text{rad/s})\sin(20^\circ)\\ &= -50\text{W} + 50\text{W}\\ &= 0\text{W}\end{aligned}$$

Yes!

Problem 5. A device called a *railgun* uses the magnetic force on currents to launch projectiles at very high speeds. An idealized model of a railgun is illustrated in the figure. A power supply is connected to two conducting rails. A segment of copper wire (the projectile), in a region of uniform magnetic field, slides freely on the rails. The wire has a 0.28mΩ resistance and a mass of 75kg.



(a) What will be the force on the segment at t = 0, when the switch is closed?

Immediately after the switch is closed, current will start to flow downward through the wire. This will cause a force $F_{B\,(on\ wire)}$ which will start to accelerate the wire rightward. Once the wire gets going, with velocity v, the magnetic field will exert a force $F_{B\,(on\ charges)}$ on the charges, exerting an effective emf $\xi_{eff} = B\ell v\cos\phi$ opposing the battery's emf. But in any event, this emf will be 0 at t = 0 because v = 0 initially. So KVL yields:

$$+200\text{V} - i(0.28\times10^{-3}\Omega) - \xi_{eff.} = 0$$

$$200\text{V} - i(0.28\times10^{-3}\Omega) - 0 = 0$$

$$i = \frac{200\text{V}}{0.28\times10^{-3}\Omega} = 714\text{kA}$$

And the resultant force will be:

$$\begin{aligned} F_{B(on\,wire)} &= lIB\sin\varphi \\ &= (0.25\text{m})(714\times10^{3}\text{A})(2\text{T})\sin 90^\circ \\ &= 357\,\text{kN} \end{aligned}$$

(b) What will be its terminal velocity?

As it speeds up, the ξ_{eff} will get larger and larger. The net emf will thus get smaller and smaller. Thus the current will get smaller and smaller. Thus the magnetic force will get smaller and smaller. Once $\xi_{eff} = \xi_{bat}$, current will cease, and the magnetic field force will vanish, and the projectile will coast at constant speed (assuming friction in the rails is negligible). So the terminal velocity will occur when,

$$\xi_{batt} = \xi_{eff}$$

$$200\text{V} = Blv\cos\varphi$$

$$200\text{V} = (2\text{T})(0.25\text{m})v\cos 0^\circ$$

$$v = \frac{200\text{V}}{(2\text{T})(0.25\text{m})} = 400\text{m/s}$$

$$\tau = mgR \to NiAB\sin\varphi = mgR \to i = \frac{mgR}{NAB\sin\varphi}$$

$$\xi_{batt} - \xi_{eff} - ir = 0 \to \xi_{batt} = NBA\omega\sin\varphi + \left(\frac{mgR}{NAB\sin\varphi}\right)r$$

$$P_{\text{sup}} = i\xi_{batt} = \left(\frac{mgR}{NAB\sin\varphi}\right)\left[NBA\omega\sin\varphi + \left(\frac{mgR}{NAB\sin\varphi}\right)r\right] = mg\omega R + \left(\frac{mgR}{NAB\sin\varphi}\right)^2 r$$

$$P_{diss} = i^2 r = \left(\frac{mgR}{NAB\sin\varphi}\right)^2 r$$

$$P_{mech} = mgR\omega$$